Location, Scale and Tail Shape Analysis of the Nigerian Stock Market Returns

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ABSTRACT

In this paper, the tail behaviour of the Nigerian All Share Index (ASI) and the stability of the distribution of the return series across different regimes were investigated. The study employed the existing classical techniques (mean, standard deviation and Fisher's measure of skewness), the quantile-based approach (median, interquartile range and Hinkley quantile skewness estimator), and pairwise-based methods (Hodges-Lehmann HL location estimator, Rousseeuw-Croux Qn statistic and medcouple MC skewness estimator) to estimate location, scale and skewness parameters of Nigerian ASI returns, respectively. The results obtained showed that the Nigerian ASI centred on zero, skewed to the left (negatively skewed) and leptokurtic (more peaked than normal). The ratio of Mean Absolute Deviation to Standard Deviation (MAD/SD ratio) and Hill's estimator were used to study the tail properties of ASI returns. The tail exponents of 1 and 2 were obtained for the left and right tails, respectively, implying that ASI returns exhibited a fatter tail than that of the normal distribution. The study concluded that the All Share Index in Nigeria is better modelled using non-Gaussian distributions as the moments around the returns exist beyond the second moment. For investors, the implication of this result is that they can gain or lose beyond their calculated expectations.

Keywords: All Share Index, Normal Distribution, tail exponent, MAD/SD ratio, Hill's Estimator

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1 Introduction

The basis of many finance analysis, such as portfolio selection, asset pricing and risk management, previously rested on the assumption of normal distribution. The popularity of this assumption in modelling financial market returns spanned several decades because of tractability and computational simplicity, among others. Also, it is supported by the Central Limit Theorem (CLT) and therefore offers the best approximation to the empirical distributions in samples of reasonable size (Mills & Markellos, 2011).

The rationale behind this prevalent view was promoted by Bachelier (1964). It was set out as follows: If the log-price changes from transaction to transaction are independently and identically distributed with finite variance, and if the number of transactions is fairly uniformly distributed in time, then applying the central limit theorem (CLT), the return distribution over large intervals such as a day, a week, or a month approaches a Gaussian shape.

However, overwhelming theoretical and empirical evidence has recently invalidated the normality assumption. Empirical analysis of asset return distribution has shown that such distributions, though unimodal and approximately symmetrical, are mainly characterized by heavy tails, high peakedness (excess kurtosis) and skewness (Rachev et al, 2005). The implication of this is that extreme values of returns are more likely than would be predicted by the normal distribution. In other words, Gaussian distribution tends to underestimate the weight of the extreme returns contained in the distribution tails (Longin, 2005). Mandelbrot (1963) was probably the first to emphasise this point. He vehemently rejected normality as distributional model for asset returns. Examining various time series on commodity returns and interest rates, he concluded that financial returns are better described by a non-normal stable distribution.

An alternative class of non-Gaussian stable distributions, also known as stable Paretian or Pareto-Levy or Levy stable distributions, was first proposed by Mandelbrot (1963a; 1963b) to model the fat-tailed nature of stock returns. The most notable extension of his work is Fama (1965), which led to the stable Paretian hypothesis. Some extensions focused on other distributions capable of modelling fat tails, such as student *t*-distribution, and hyperbolic distributions. All these extensions have greatly contributed to understanding the distributional behaviour of asset returns. Other studies that have affirmed the usage of stable distributions in modelling financial or related data are Rachev & Mittnik (2000) and Kim et al (2011).

Recently, Bekri & Kim (2014) investigated portfolio management in Islamic finance. The study provided empirical evidence that assets in Islamic finance exhibit asymmetry, heavy tails and volatility clustering.

Empirical investigation of the tail behaviour of financial market returns distributions, assessment of how fat-tailed returns are and evaluation of the stability of the returns distributions across different regimes have received the attention of researchers because of the developments in statistics and econometrics. For instance, Koedijk & Kool (1992) used a nonparametric tail-index estimator based on extreme-value theory to shed light on some of the characteristics of the empirical distribution of black-market exchange-rate returns for seven East European currencies between 1955 and 1990. Other similar studies include Koedijk et al (1990), Hols & de Vries (1991) and Loretan & Phillips (1994).

Empirical examination of the similarities between the left and right tails of return distribution has also been conducted. A typical example in this case is Jondeau & Rockinger (2003). The authors investigate whether the perception that the left tails are heavier than the right ones is due to clustering of extremes. The finding shows that, the left tail of the returns is similar to the right tail and concluded that volatility clustering cannot be held responsible for this perception.

Following the above, what is the current state of knowledge about the behaviour of the Nigerian stock market returns? What is the shape of the distribution, Gaussian or non-Gaussian? What is the tail shape characteristic? The need to update the existing knowledge about the tail characteristics or behaviour of financial market returns will continue to be of interest in finance literature on Nigeria. Consequently, the specific objective of this study is to determine the shape of the Nigerian ASI and analyze its tail characteristics. The paper examined the information in the tails of the distribution of the stock market returns. Precisely, it examined the left and right tail shapes of the empirical distributions of the returns. Given that stock returns are not Gaussian, the properties of the unconditional distribution are important, and distribution tails are particularly interesting. LeBaron (2008, page 2) gives the following reasons why measures of tail properties are important:

> First, for researchers calibrating to these features, they give them a quantitative target which is more challenging and interesting than simply getting non-normal return distributions. Second, the shape of the tail parameter gives us important information about the existence of higher moments in return series. Unstable, or non-existent, higher moments can cause problems for estimating other parameters, or various measures of risk. Third, estimates of tail shape can be used for better risk estimation, since they provide information on tail probabilities. Finally, tail shape also can connect various risk measures as in expected tail loss and VaR.

The paper is planned as follows. After this introductory section, a brief review of literature on extreme value theory is conducted in section 2. In section 3, the methodology employed in the tail shape analysis is discussed while section 4 presents the analysis and the Monte Carlo experiment used to determine the appropriate number of tail observations for estimating the tail index. The summary and conclusion of the paper is given in section 5.

2 Review of Literature

According to Martin and Jan (2006), Extreme Value Theory (EVT) is a useful substitute to the traditional Value-at-Risk (VaR) method for measuring risk exposure. Although VaR has been established as a standard tool among financial institutions to depict the downside risk of a market portfolio, it has been shown by Jorion (1997) that due to risky market factors, it only measures the maximum loss of the portfolio value over some period at some specific confidence level. Also the standard VaR method such as variance-covariance method or historical simulation can fail when the return distribution is fat tailed. This problem is aggravated when long term VaR forecasts are desired.

After the seminal work of Markowitz on portfolio theory, volatility has become an extremely important variable in finance, appearing regularly in models of asset pricing, portfolio theory, risk management, etc. (Mills & Markellos, 2011). Various measures of volatility have been developed and the most common is the unconditional standard deviation of the historical returns. The limitations of this method have led to other measures such as the semi-variance by Nawrocki (1999) and the absolute deviation by Granger and Ding (1995).

In addition to the historical standard deviation, several extreme value estimators have been suggested in the literature in an effort to improve efficiency using information contained in the opening, closing, high and low prices during the trading day. For instance, Garman & Klass (1980) propose minimum variance unbiased extreme-value estimator. The integrated or realized variance non-parametric estimator has also become very popular over the past decade after a series of papers by various authors such as Barndorff-Nielsen et al (2004) and Anderson et al (2007).

An alternative approach to measuring volatility is to incorporate it within a formal stochastic model for the time series itself. This is usually accomplished by allowing the variance (or conditional variance) of the process generating the time series to change either at certain discrete point in time or continuously. For instance, Harvey & Shephard (1996) and Jacquier et al (2004) have developed two different models which allow for correlation between the shocks in the mean and variance processes.

In concluding this review, it is important to note that there are challenges in the implementation of extreme value theory. According to Younes (2000), these include the paucity of extreme data, problem of determining whether the series is "fat-tailed," choosing the threshold or beginning of the tail, and choosing the methods of estimating the parameters. For these reasons, paying attention to the challenges above while investigating tail behavior is very important.

3 Methodology

3.1 Extreme Value Theory and the Tail Index Estimator

Consider θ_1, $\theta_2,\ldots,\theta_n$ to be a stationary sequence of independent identical distribution (iid) All Share Index (ASI) with distribution function F(*). Define Max_n as the maximum of this sequence of indexes: Then,

$$Max_n = \max(\theta_1, \theta_2, \ldots, \theta_n). \quad (1)$$

It may therefore be shown that the distribution function $F^n(x)$ of Max_n for a large n converges towards the same limiting distribution H(x), independent of whether the ASI were generated by a Student-t or some stable distribution. As the competing distributions are hence nested within the same limit law H(x), there is no need to specify or maintain hypothesis about the correct F(x).

The limiting distribution H(x) is of the following form, with β > 0 and the tail index a = 1/ β:

$$H(x) = 0, \qquad x < 0$$
$$H(x) = \exp(-x)^{-1/\beta} = \exp(-x)^{-\alpha},\ x \geq 0 \qquad (2)$$

Leadbetter et al. (1983) showed that the theory also holds in a situation where that assumption of independence for the ASI is inappropriate, as long as the dependency is not too strong. Likewise, for the family of symmetric stable Paretian distributions, the tail index *a* in equation (2) may be interpreted as the characteristic exponent of the stable distribution, which ranges between (0, 2). Approximately, the lower the value of *a*, the thicker are the tails of the distribution, ceteris paribus. Also, for the class of Student-*t* distributions, the tail index *a* in equation (2) is the number of degrees of freedom of the distribution, which ranges from 0 to ∞.

Thus, the simple and efficient estimator of the tail index is given as:

$$\widehat{\beta} = \frac{1}{\widehat{\alpha}} = \frac{1}{m}\sum_{i=1}^{m}[log\theta_{(n+1-i)} - log\theta_{n-m}] \qquad (3)$$

Where *n* represents the total number of ASI observations and *m* is the number of tail observations used to estimate a. Equation (3) is based on Hill (1975) and Mason (1982) established that under some basic regularity condition, $\widehat{\beta}$ is a consistent estimator for β. Similarly, Goldie and Smith (1987) proved that $\widehat{(\beta - \beta\ )}m^{1/2}$ is asymptotically normal with mean 0 and variance β^2. As a result, $\widehat{\alpha}$ is also asymptotically normal with mean a and variance $\frac{\widehat{\alpha}^2}{m}$, and asymptotic confidence intervals may be constructed to test specific hypotheses. The work of Koedijk et al (1990) established an empirical application of this estimator.

From the expression in equation (3), the estimator uses only the positive tail (right tail) of ASI to estimate a and neglects the content of the large negative observations in the left (negative) tail. In order to affirm that the right and left tails have the same tail index, we combine the information in the right and left tails by obtaining the absolute values of the ASI, we then order the ASI and use equation (3) to obtain the estimator. By this, the precision of the tail index may, by all instances, be improved significantly. Thus, we use the Monte Carlo simulation to derive the number of tail observations to be used. The results of our simulation are presented in Section 4.2.

3.2 Tail Shape Determination

Let S_t be the stock market index at time *t*. Empirical work on the distribution of financial returns is usually based on log returns. The continuously compounded or log return from time *t* to time $t + \Delta t$, $r_{t+\Delta t}$ is then defined as:

$$r_{t,t+\Delta t} = logS_{t+\Delta t} - logS_t \qquad (4)$$

To interpret the quantity in (4) as percentage returns, we simply multiplied it by 100. Since the horizon over which stock returns are calculated is daily, Δt can be set equal to 1. Therefore, dropping the first subscript, (4) can be written as:

$$r_t = logS_t - logS_{t-1} \tag{5}$$

The log returns (5) can be additively aggregated over time.

Varying approaches are used in characterising tail behaviour of return distributions (for example, Koedijk et al, 1990). But Loretan and Philips (1994) formalize all arguments concerning tail behaviour of return distribution by defining it to take the form:

$$P(X > x) = C^{\gamma} x^{-\gamma} (1+\gamma_R(x)) \qquad x>0$$
$$P(X < -x) = C^{\gamma} x^{-\gamma} (1+\gamma_L(x)) \qquad x>0,$$

Where C and γ (tail index) are parameters estimated using order statistics. $\gamma_R(x)$ and $\gamma_L(x)$ are information sets about the right and left tails, respectively.

Although several approaches have been developed to estimate tail index, the Hill's estimator (Hill, 1975) is mostly adopted because it is simple and efficient. As shown in equation 3, above.

3.3 Location Estimators

Measures of location are very important in statistics. Apart from providing a summary account of the data, they tend to help better understand the data. The three Ms (mean, median and mode) appear to be the most common measures of location but the mode has received lesser attention because of its inability to elicit interesting information to researchers. The other two have been extensively used in practice and have been seen to complement each other.

A less common pairwise estimator, known as the Hodges-Lehmann midpoint estimator (Hodges and Lehmann, 1963) is given below:

$$HL = med\{\frac{x_i+x_j}{2}; i < j\}, \tag{6}$$

Equation (6) has been shown to outperform the mean in terms of robustness (Gelade et al., 2015). But owing to its computational complexity, this estimator has not been extensively used. Although Gelade et al. (2015) developed STATA codes to cater for this estimator and even reduced its computational time using an efficient algorithm proposed by Johnson and Mizoguchi (1978), in this paper, an alternative R script capable of performing similar analysis was written for the computation of this particular estimator.

3.4 Scale Estimators

The classical sample standard deviation (s = $\left(\sum_{i=0}^{n} \frac{(\mathrm{X}-\bar{\mathrm{X}})^2}{\mathrm{n}-1}\right)^{\frac{1}{2}}$) has been the most popular of all scale estimators. It is a very efficient estimator of the population standard deviation σ when using Gaussian data but it lacks robustness like the mean.

Although quantile-based estimators like the Median Absolute Deviation ($MAD = b \times med_i\left|x_i - x_j\right|$, $b = 1.4826$) and the Interquartile Range ($IQR = d \times Q_{0.75} - Q_{0.25}$) $d = 0.7413$, have been used as robust alternatives to estimating the standard deviation, they have been shown to exhibit one fault or the other (Gelade et al., 2015). A less known, difficult to use but rather robust and efficient scale estimator is the Q_n statistic proposed by Rousseeuw and Croux (1993). This statistic

$$Q_n = d \times (\left|X_i - X_j\right|; i < j) \tag{7}$$

Outperforms the IQR and MAD in terms of robustness and has been shown to be consistent for Gaussian data as well. Although it can be easily implemented when the data to be used is small in size, it can be time-consuming and computationally complex for very large data set. An R script was also written for the computation of this estimator.

3.5 Skewness Estimators

The most commonly used estimator is the Fisher estimator ($\gamma = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{x_i - x_j}{s}\right\}^3$) but because it is an estimator that relies on the mean and standard deviation, it shares their demerits. It performs poorly in the presence of extreme values because of its non-resistance to outliers. Other skewness-based estimators, such as $\frac{\bar{x} - mode}{s}$ and $\frac{\bar{x} - Q_{0.5}}{s}$, have been proposed by Pearson (1916) which still share the same demerits as the Fisher's skewness.

The quantile-based estimator of skewness first proposed by Yule and Kendall (1968) and later generalized by Hinkley (1975) is a very much robust alternative. The estimator, which is given as:

$$SK_p = \frac{(Q_{1-p} - Q_{0.5}) - (Q_{0.5} - Q_p)}{Q_{1-p} - Q_p} \; where \; 0 < p < 0.5 \tag{8}$$

is very robust to outliers if p is set to 0.25. Another alternative robust SK operator is called the medcouple (MC) proposed by Brys, Hubert and Struf (2004). It is a pairwise-based estimator and replaces all quantile points in (8) with actual data points. It is given as

$$MC = med_{x_{(i)} \leq Q_{0.5} \leq x_{(j)}} h(x_{(i)}, x_{(j)}) \text{ for all } x_i \text{ and } x_i \neq x_j \tag{9}$$

The kernel function *h* is given by

$$h(x_{(i)}, x_{(j)}) = \frac{(x_{(j)} - Q_{0.5}) - (Q_{0.5} - x_{(i)})}{x_{(j)} - x_{(i)}} \qquad (10)$$

3.6 Data, Sources and Description

The data used in this study was the daily All Share Index of the Nigerian Stock Exchange from 3rd January, 2006 to 18th July, 2016. All days without trading were omitted, thus leaving us with data for days with trading activities. In all, 2,599 observations were used. The entire dataset was downloaded from the official website of the Nigerian Stock Exchange (www.nse.com.ng).

4 Results and Analysis

4.1 Preliminary Analysis

4.1.1 Descriptive Account of ASI and its Returns

This section gives an overview and descriptive account of the data in comparison to known properties of the normal distribution. The figure below presents a line plot for the series. The plot shows a rise in the first 500 observations followed by a decline but an upward trend is noticed from the 1000th observation. Figure 2 shows the line plot for returns (computed based on equation 5 above); though the series can be said to centre on 0, the volatility of the series is high with cases of extreme values.

Figure 1: The Nigerian All Share Index between January 2006 and July 2016

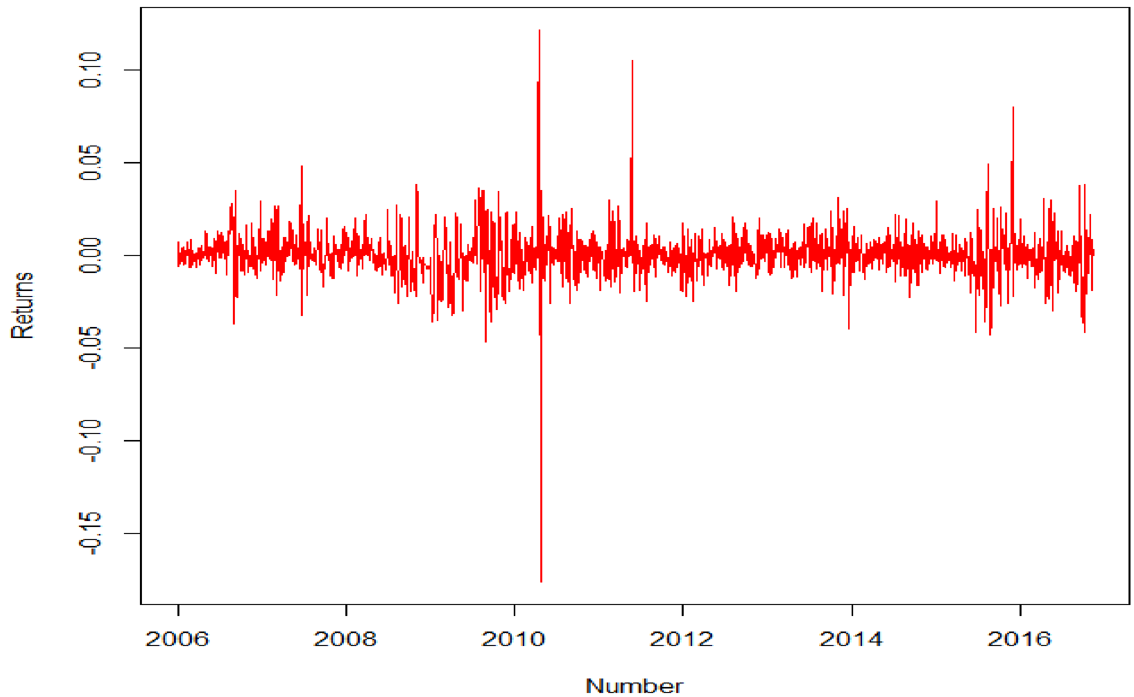

Figure 2: Returns for Nigeria All Share Index between January 2006 and July 2016

The histogram and density plot (see Figure 3a) for returns shows that the empirical return distribution of ASI is highly peaked compared to the theoretical normal distribution with the same mean and standard deviation. And Figure 3b (box plot) also revealed the volatilities in ASI. Also, the tail for return distribution is thicker than that of the normal distribution and it has excess kurtosis. These features depict heavy-tailed distribution. The Normal QQ-Plot (Figure 4) below further justifies this conclusion because the computed theoretical quantiles deviate from the line at both ends.

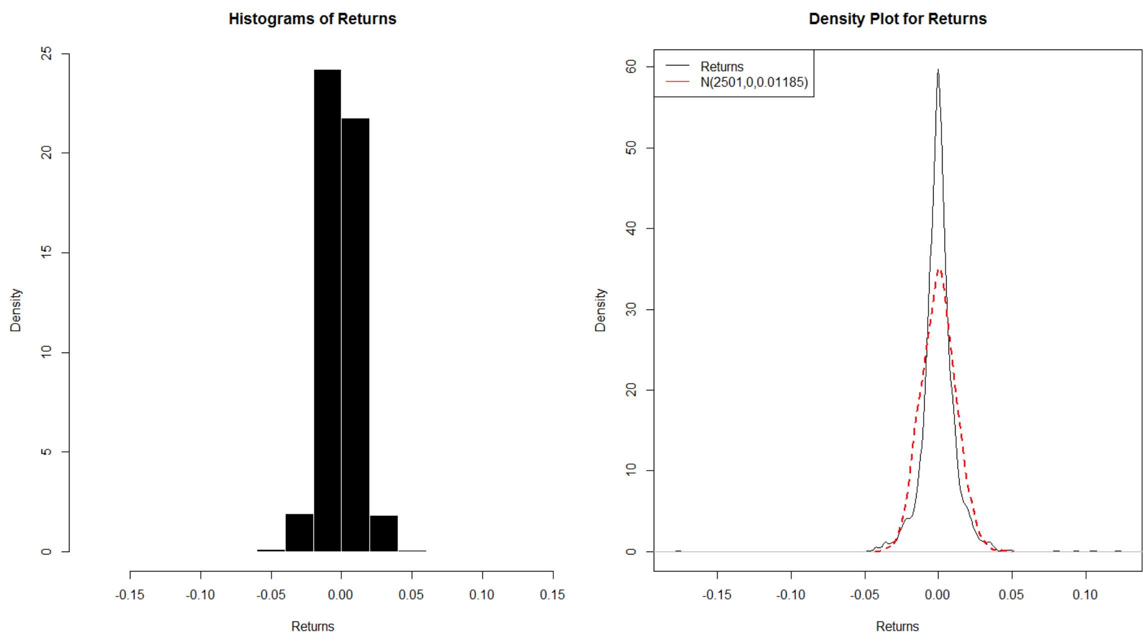

Figure 3a: Returns Histogram and Density Plot for Return Distribution

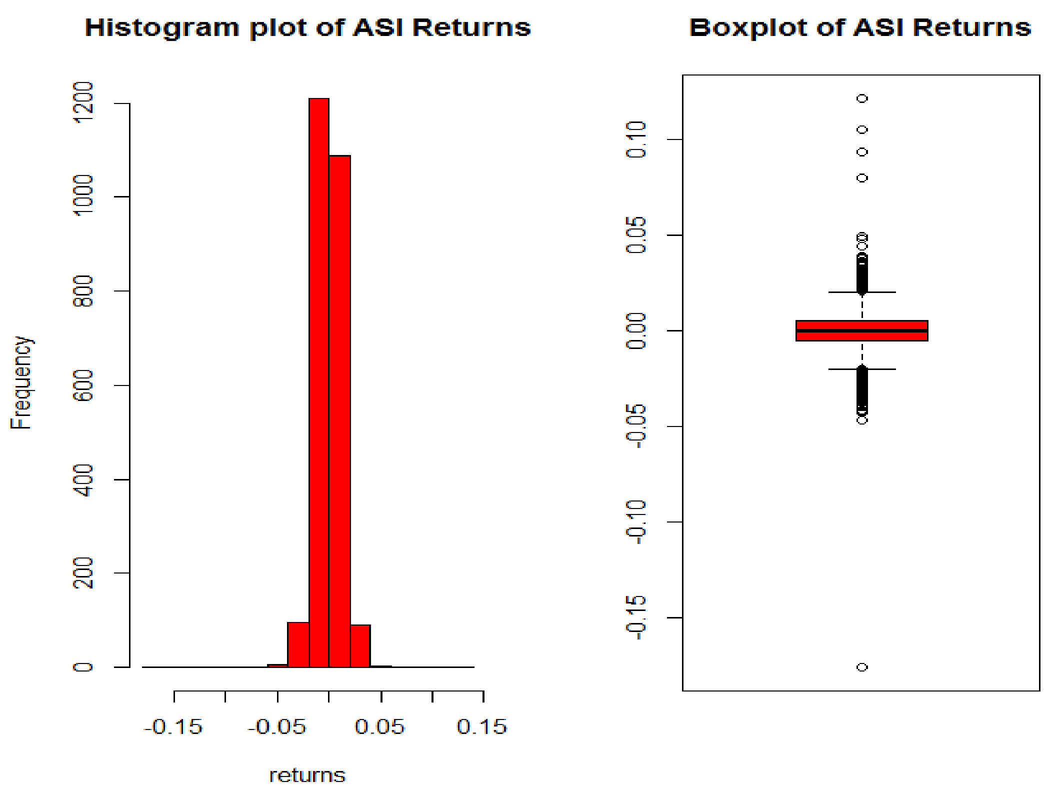

Figure 3b: Returns Histogram and Box Plot for Return Distribution

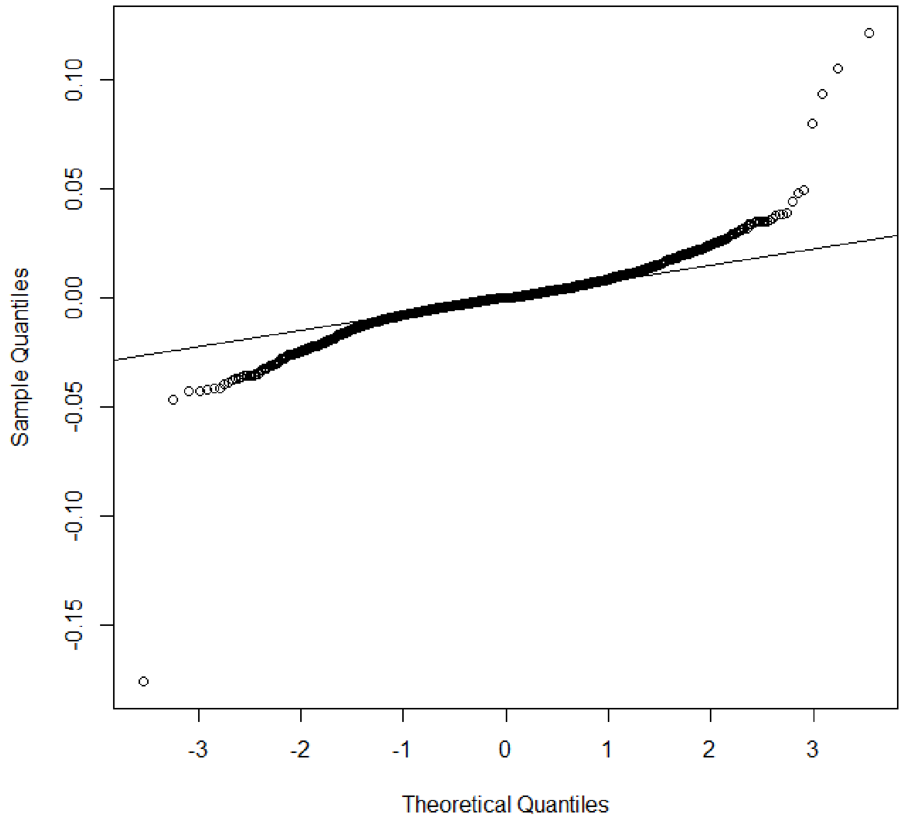

Figure 4: QQ-Plot for Return Distribution

To give a proper descriptive account of ASI returns, for each of location, scale, and skewness estimators, classical, quantile-based and pairwise-based approaches were used. This was done in order to obtain robust estimates against outliers.

4.1.2 Location Estimators of Returns of ASI

In location analysis, the mean is very efficient with Gaussian data but unreliable in the presence of outlier. That is, mean is affected by extreme values, hence meaningless with highly asymmetric data. However, when data is non-Gaussian, HL is more efficient. As shown in Table 1, all computed location estimators show that the distribution of returns of ASI is centred on zero (0).

Table 1: Location Estimates for ASI Returns

Estimator	Type	Estimate
Mean	Classical	5.570e-06
Median	Quantile-based	-3.250e-06
HL	Pairwise	-8.3e-05

Source: Computed by the Authors. **HL**: Hodges-Lehmann estimator (eq. 6)

4.1.3 Scale Estimators of Returns of ASI

In estimating the scale parameter, the classical estimator is the standard deviation (SD) which is the most efficient estimator when the data is Gaussian, though it is not robust to outliers. However, when dealing with non Gaussian series like ASI, the alternative robust estimators with efficiency and robustness properties are interquartile range (IQR) and Q_n Statistics of Rousseeuw and Croux (1993). The results of these statistics are shown in Table 2. Each of the scale estimators shows that the spread of ASI returns is not large and is highly clustered around the central value.

Table 2: Scale Estimates for ASI Returns

Estimator	Type	Estimate
S	Classical	0.011853
IQR	Quantile-based	0.0078837
Q_n	Pairwise	0.0122049

Source: Computed by the authors. **Qn**: Rousseeuw-Croux Scale Estimator (eq. 7)

4.1.4 Skewness Estimators of Returns of ASI

The widely used skewness estimator is the Fishers' estimator. However, it has no resistance to outliers since it relies on mean and standard deviation (SD). Thus, $SK_{0.25}$ and MedCouple (MC) are the alternative skewness estimators to Fisher. In this analysis, the three estimators provide the same information about the direction of the tail of the distribution of the ASI index. The results in Table 3 showed that ASI is negatively skewed implying that the longest tail is to the left.

Table 3: Skewness Estimates for ASI Returns

Estimator	Type	Estimate
Fisher	Classical	-0.34855
$SK_{0.25}$	Quantile based	-0.003370
MC	Pairwise	-0.061583

Source: Computed by the authors **MC**: Medcouple skewness estimator (eq. 9)

4.1.5 Measuring Fatness of Tail Distribution

The Pareto Tail Index method is mostly used to measure the fatness of tail of return distributions. This method, however, has a major demerit because it is a "curve fitting" approach, where you start by assuming a particular distribution, then see which parameter gives the best fit.

For this study, a comparison between the Mean Absolute Deviation (MAD) and Standard Deviation (SD) in terms of ratio to gauge fatness of tail was employed. The MAD is without squares unlike the standard deviation, which makes it less volatile to outliers. Also, the MAD/SD ratio cannot exceed 1. Therefore, the closer the ratio is to 1, the fatter the tails of the distribution in question. The figures below show the MAD/SD ratio for the return distribution of ASI compared to a standard normal distribution and the student t distribution. The first plot was generated using n = 200 (Figure 5), then n = 300 (Figure 6) and n = 500 (Figure 7). Visual inspection of the plots below shows that, at any degrees of freedom, for any sample size, the MAD/SD ratio for the return distribution of ASI does not exceed 0.7, while that of the normal distribution is consistently above 0.7. Precisely, the difference between the MAD/SD ratio of the normal distribution and ASI returns distribution is approximately 1.5. This further corroborates the graphical results indicating the fatness of the tails. The phenomenon of fat tail implies that there is a probability, which may be small, that an investment will move beyond three standard deviations. This is often referred to in literature as the concept of tail risk. For a normal distribution, the probability that returns will move between the mean and three standard deviations, either positively or negatively, is approximately 99.97%. The implication of this is that the probability of returns moving more than three standard deviations beyond the mean is 0.03%. However, the MAD/SD ratio for student t distribution tends to converge towards the MAD/SD ratio for ASI; and the degrees of freedom for the MAD/SD ratio for both the student t and ASI converges between 1 and 3. This is, however, crude and might not be a very accurate estimate of tail exponent; thus, the formalization of tail behaviour as suggested by Loretan and Philips (1994) using the Hill's estimator was exploited.

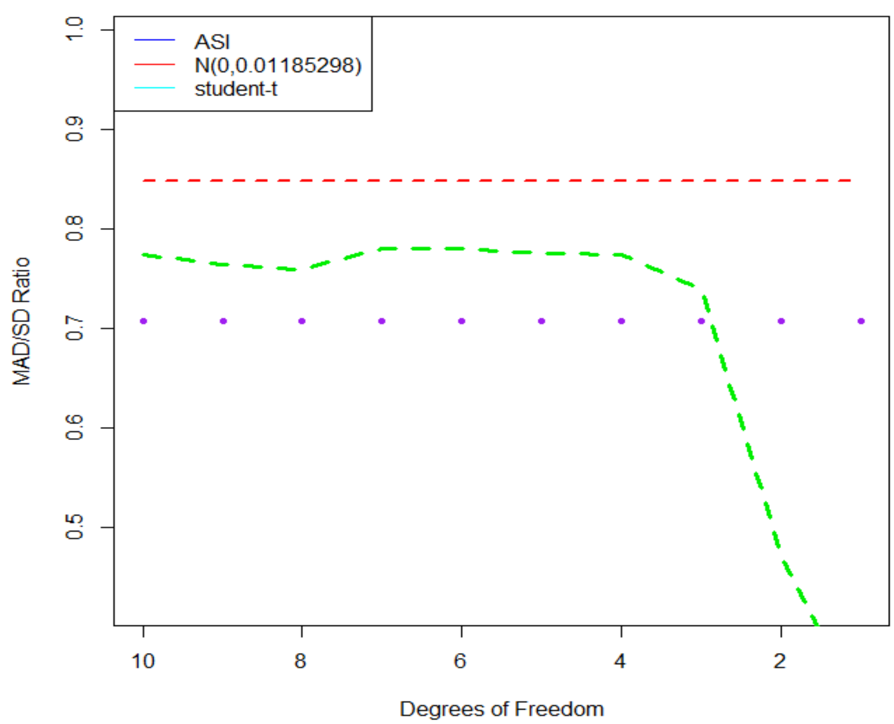

Figure 5: MAD/SD Ratio using n=200

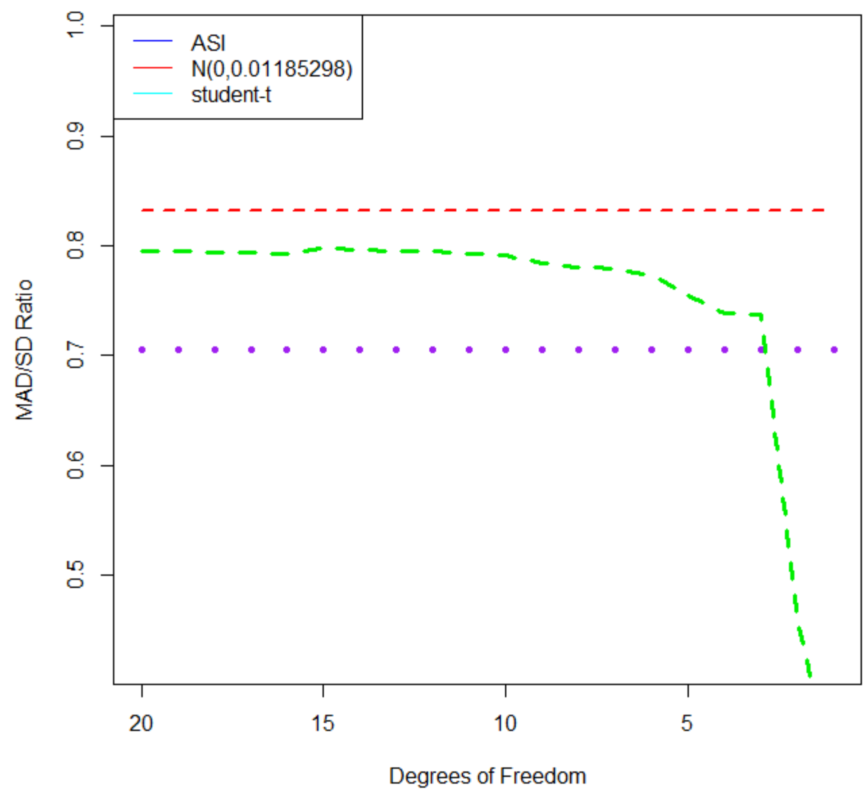

Figure 6: MAD/SD Ratio using n=300

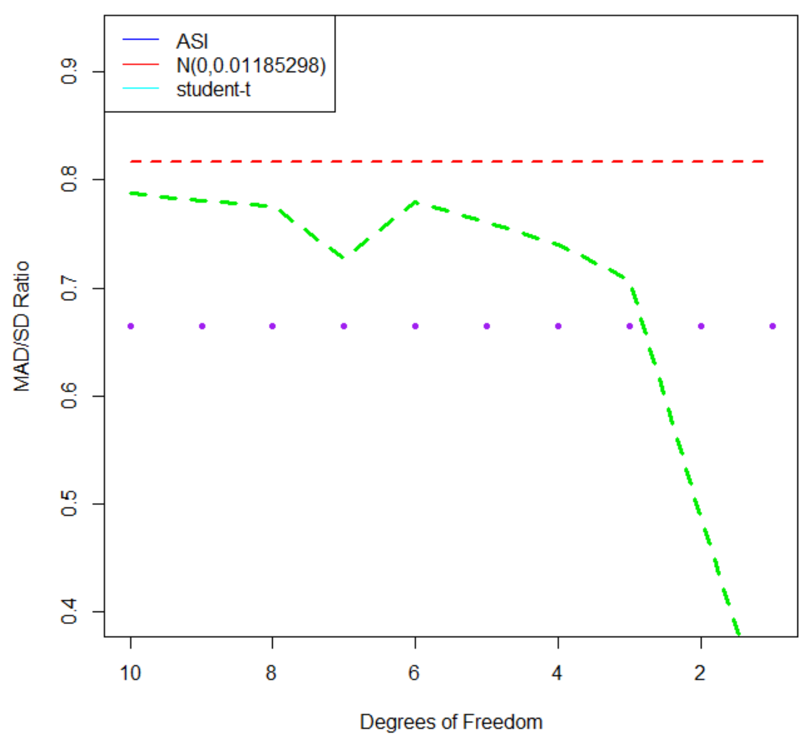

Figure 7: MAD/SD Ratio using n=500

4.2 Monte Carlo Experiment: Choosing the Optimal M Level

Although the MAD/SD ratio gave a rough estimate of the tail index, a Monte Carlo experiment was carried out to obtain the optimal m level to compute the tail index using the Hill's estimator. The Monte Carlo simulation experiment is very much like that carried out by Koedijk and de Vries (1990) but it is much more precise and direct since the MAD/SD ratio has given an idea of where the tail index will possibly lie. Unlike Koedijk and de Vries (1990), the scope of the Monte Carlo experiment was limited to simulating from t distributions with degrees of freedom 1, 2 and 3 and an upper bound of 200 was placed on m since the value of m must not exceed 0.1T, where T is the length of the whole series (Loretan and Philips, 1994). The results of the optimal m level resulting from the Monte Carlo simulation, estimated tail index and MSE are reported in Table 4.

Minimum MSE for tail index occurs at m = 13 for α = 1; 189 for α = 2; and 8 for α = 3 for right tail. For the left tail, minimum MSE values were obtained for m = 140 for α = 1; 16 for α = 2; and 57 for α = 3. One important point to note here is that across the right tail α = 2 possess the smallest MSE, while α = 1 has the smallest MSE value for the left tail.

Table 4: Optimal Choice of m through Monte Carlo

Degrees of freedom		**α** = 1	**α** = 2	**α** = 3
	Optimal	**m = 13**	**m = 189**	**m = 8**
Right tail only	**Tail Index**	1.002786	2.00041	2.948918
	MSE	489836e-07	1.186523e-08	0.000185
	Optimal	**m = 140**	**m = 16**	**m = 57**
Left tail only	**Tail Index**	1.000119	1.802584	2.999196
	MSE	9.974349e-10	0.002755816	4.568261e-08

Source: *Computed by the authors.* ***Tail Index:*** *Hill Estimator (eq. `3).* ***Key:*** *MSE – Mean Square Errors*

4.3 Discussion of Results

Literature provides alternative criteria such as minimum variance and smallest bias for choosing the appropriate tail index (α, degrees of freedom in this case) but we prefer to use the smallest Mean Square Error (MSE) associated with the optimal m levels in computing information about the left and right tails based on the Hill's estimator. The use of MSE is justified because it measures the differences between the observed and estimated values. More importantly, MSE is the second moment (about the origin) of the error, and thus incorporates both the variance of the estimator and its bias. For an unbiased estimator, the MSE is always the variance of the estimator.

Table 4 indicates the appropriate tail index was α = 1 for the left tail and α = 2 for the right, since the MSE was significantly lower than others. This was expected because the MAD/SD plots indicated that the tail index is in the neighbourhood of 1 and 3, since the

Student-t and the returns of ASI intersected in this region. Also, the methodology employed appears to be robust to choice of m as standard errors of estimates for the tail indices estimated are very small at any level of m except m = 1 for both left and right tails.

These findings point to the fact that returns from the ASI cannot be modelled using the normal distribution despite it being centred around zero. Its tail was fatter than that of the normal distribution and it had a tail index $\alpha = 1$ and $\alpha = 2$ for the left and right tails, respectively. The implication of this result is that ASI returns have kurtosis that exceeds that of normal distribution, indicating the occurrence of returns on investment beyond what could be expected. More precisely, unlike the predictions of the normal distribution, the results show that the stock market has experienced returns that exceeded three standard deviations beyond the mean in more than 0.03% of the observations.

Comparatively, the Nigerian All Share Index returns had tail indices lying in the region $1 \leq \alpha \leq 2$, which implies that all moments around it are finite compared to the findings of Jansen and de Vries (1991), Loretan and Phillips (1994) and de Haan et al. (1994), who all estimated the tail indices for US stock and bond market returns as lying in the region $2 < \alpha < 4$. However, the findings of Koedijk et al (1990), Koedijk, Stork and de Vries (1990), Hols and de Vries (1991), and Koedjik and Kool (1992), who all estimated the tail indices of foreign exchange rate returns in European black market as falling in the region $1 \leq \alpha \leq 2$. Similarly, recent study on US daily stock returns by LeBaron (2008) found the scaling exponents to be around 3 and to be generally stable over time and across positive and negative tails. Others such as Bekri and Kim (2014) provided similar empirical evidence to show that Islamic finance assets returns showed asymmetry, heavy-tail and volatility clustering and therefore suggested the use of stable distributions and the student's t related copulas for portfolio modelling.

5 Summary and Conclusion

This paper reviewed the returns of the ASI, explored and obtained its descriptive properties and described its tail properties by estimating its tail index based on extreme value theory. A ratio of the Mean Absolute Deviation and Standard Deviation was used to characterise tail properties of the return distribution of ASI, the normal distribution and the student t distribution across several degrees of freedom, which helped pinpoint where the tail index of the return distribution of ASI could possibly lie. The Hill's estimator was employed in obtaining the tail index but a Monte Carlo simulation approach was used in obtaining the major issue about the Hill's estimator (m).

The results suggested that the return distribution of ASI was negative but the kurtosis value showed that the data was highly peaked. The MAD/SD ratio indicated a student t distribution with degrees of freedom between 1 and 2, both inclusive. This result was further confirmed by the Monte Carlo simulation carried out to obtain the optimal level of m in the Hills estimator, which helped confirm that the appropriate tail indices for left and right tails of the return series for ASI are 1 and 2, respectively.

The crux of this paper is to establish the impact of tails distribution in the returns of the Nigerian ASI. As supported by our results, the ASI is not Gaussian. Therefore, investors should understand that asymmetry in stock market returns is possible as returns are based on the tail distribution. Indeed, this finding is a major characteristic of stock market returns. According to LeBaron and Samanta (2004), equity market crashes or booms are extreme realizations of the underlying return distribution. Hence, it is important for the market operators to continuously monitor the tail behaviour where crashes and booms are generally reflected.

References

Anderson, T., Bollerslev, T., & Diebold, F. (2007). Parametric and NonParametric Volatility Measurement, in Y. Ait-Sahalia & L.P. Hansen (eds.), *Handbook of Financial Econometrics,* New York: Elsevier.

Bachelier, L. (1964). Theory of Speculation, in P.H. Cootner (ed): *The Random Character of Stock Market Prices*. MIT Press. pp. 17-75.

Barndorff-Neilsen, O., Graversen, S., & Shephard, N., (2004). Power Variation and Stochastic Volatility: a Review and some New Results, *Journal of Applied Probability,* 41, 133-143.

Bekri, M., & Kim, A. (2014). Portfolio Management with Heavy-Tailed Distributions in Islamic Finance. *Journal of Islamic Economics, Banking and Finance (JIEBF),* 10(2)

Brys, G. M., Struf, A. (2004). A Robust Measure of Skewness. *Journal of Computational and Graphical Statistics* 13: 996-1017.

De Haan, L., Jansen, D. W., Koedijk, K., & de Vries, C. G. (1994). Safety first portfolio selection, extreme value theory and long run asset risks. In *Extreme value theory and applications* (pp. 471-487). Springer US.

Fama, E., (1965). The Behaviour of Stock Market Prices. *Journal of Business*. vol. 38: 34-105.

Garma, M. & Klass, M., (1980), On the Estimation of Security Price Volatilities from Historical Data, *Journal of Business,* 53, 67-78.

Gelade, W., Verardi, V., & Vermandele, C. (2015). Time-efficient algorithms for robust estimators of location, scale, symmetry, and tail heaviness. *Stata Journal*, *15*(1), 77-94.

Goldie, C., & Smith, R. (1987). Slow Variation with Remedies Theory and Application, *Quarterly Journal of Mathematics,* 38, 45 – 71.

Granger, C. & Ding, Z. (1995). Some Properties of Absolute Returns: An Alternative measure of Risk, *Annales d'Economie et de Statistique,* 40, 67-91.

Harvey, A., & Shephard, N. (1996). Estimation of an Asymmetric Stochastics

Hill, B. M. (1975). A Simple Approach to Inference about the Tail of a Distribution. *The Annals of Statistics,* 3: 1163-1174.

Hinkley, D. V. (1975). On Power Transformation to Symmetry. *Biometrika* 62: 101- 111.

Hodges, J. L., and Lehman, E. L. (1963), Estimates of Location Based on Rank Tests. *Annals of Mathematical Statistics.* 34: 598- 611.

Hols, M.C., and De Vries, C. G. (1991), The Limiting Distribution of Extremal Exchange Rate Returns, *Journal of Applied Econometrics*. 6: 287-302.

Jacquier, E., Polson, N., and Rossi, P. (2004). Bayesian Analysis of Stochastic Volatility Models with Fat-tails and Correlated Errors, *Journals of Econometrics,* 122, 185-212.

Jansen, D. W., and de Vries, C. G. (1991). On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective. *Review of Economic and Statistics.* 73, 18- 24.

Johnson, D. B., and Mizoguchi, T. (1978), Selecting the Kth Element in X+Y and $X_1 + X_2 + ... + X_m$. *SIAM Journal on Scientific Computing* 7: 147-153.

Jondeau, E. and Rockinger, M (2003), Testing for Differences in the Tails of Stock-Market Returns, *Journal of Empirical Finance*, 10: 559-581.

Jorion, P. (1997): Value at Risk –The New Benchmark for Controlling Market Risk. McGraw-Hill, New York.

Kim, Y. S., Rachev, S. T., Bianchi, M. L., Mitov, I., & Fabozzi, F. J. (2011). Time series analysis for financial market meltdowns. *Journal of Banking & Finance*, *35*(8), 1879-1891.

Koedijk, K.G., and Kool, C. J. (1992). Tail Estimates of East European Exchange Rate, *Journal of Business and Economic Statistics*. 10: 83-96.

Koedijk, K.G., Schafgans, M. M., and De Vries C. G. (1990). The Tail Index of Exchange Rates Returns. *Journal of International Economics*. 29:93-108.

Leadbetter, M., Lindgren, M., and Rootzen, H. (1983). Extremes and Related Properties of random Sequence on Processes, New-York: *Springer-Verlag.*

LeBaron, B. (2008). Robust properties of stock return tails. *International Business School, Brandeis University*.

Loretan, M. and Phillips, P.C. (1994). Testing the Covariance Stationarity of Heavy tailed Time Series: An Overview of the Theory with Applications to Several Financial Datasets. *Journal of Empirical Finance*. 1: 211-48.

Longin, F. (2005). The Choice of the Distribution of Asset Returns: How Extreme Value Theory Can Help? *Journal of Banking and Finance*. 29: 1017-1035.

Martin Odening and Jan Hinrichs (2006). Using Extreme Value Theory to Estimate Value At Risk, Retrieved from https://www.agrar.hu-berlin.de/de/institut/departments/daoe/abl/dokumente/literatur/VAREVT.

Mason, D. M. (1982). Laws of Large Numbers for Sums of Extreme Values. *Annals of Probability,* 10,:754-764.

Mandelbrot B. B. (1963a). The Variation of Certain Speculative Prices. *Journal of Business*. vol. 26, No. 4:394-419.

Mandelbrot B. B. (1963b). New Methods in Statistical Economics. *Journal of Political Economy*. 71: 421-40.

Mills T.C. and Markellos R. N (2011). *The Econometric Modelling of Financial Time Series.* Third edition. Cambridge: Cambridge University Press.

Nawrocki, D (1999). 'A brief History of Downside Risk <easures', *Journal of Investing,* 8, 9 – 25.

Rachev, S. T., Menn, C., & Fabozzi, F. J. (2005). *Fat-tailed and skewed asset return distributions: implications for risk management, portfolio selection, and option pricing* (Vol. 139). John Wiley & Sons.

Rachev, S., and Mittnik, S. (2000), *Stable Paretian Models in Finance*. In Series in Financial Economics, John Wiley & Sons.

Roussew, P. J., and Croux, C. (1993). Alternatives to the Median Absolute Deviation. *Journal of the American Statistical Association.* 88: 1273-1283.

Younes, B.(2006). Steps in Applying Extreme Value Theory to Finance, A Review, Retrieved from: http://www.banqueducanada.ca/wp-content/uploads/2010/01/wp00-20.pdf

Yule, G.V., and Kendall, M. G. (1968). *An Introduction to the Theory of Statistics.* 14th Edition*.* London: Griffin.